Economics 212

Section 002

Midterm Exam

October 25, 2011

Student Number: Answer Key

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=5X+5Y, where X and Y are two goods. Assume the price of X is \$6, the price of Y is \$12 and the consumer has an income of \$1800. Derive the optimal consumption bundle for the consumer.

The consumer gets as much utility from both substitutes. He will therefore only consume the cheaper of the two.

$X^* = 1800/6$

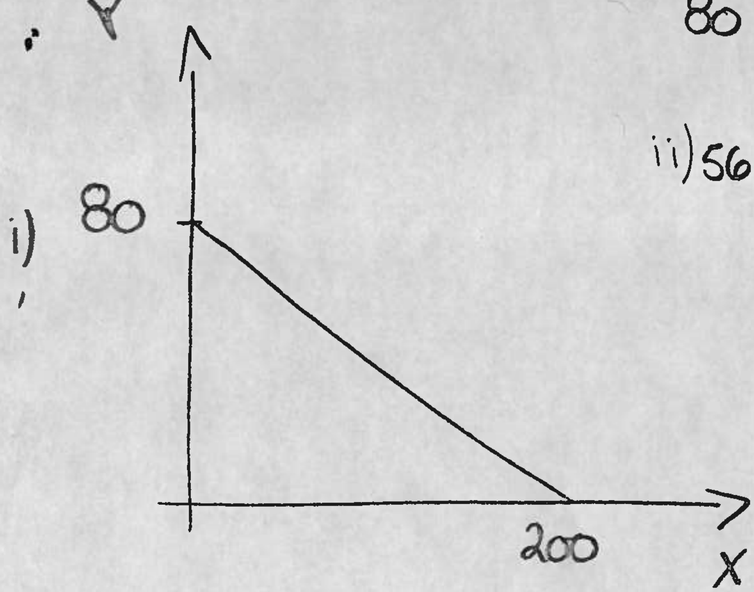
$y^* = 0$

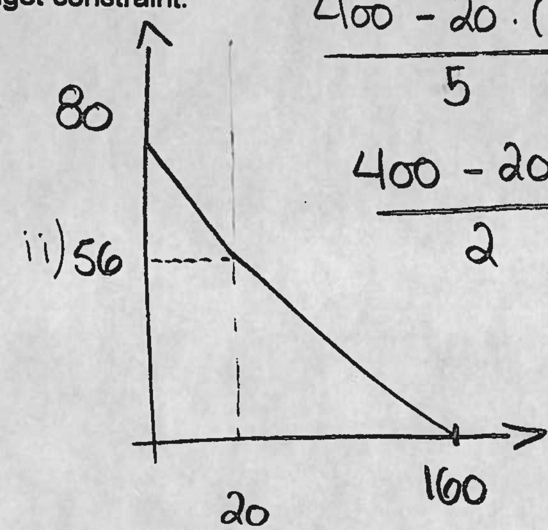
2. [5 marks] A consumer has \$400 in income and purchases two goods, X, which has a price of \$2 and Y, which has a price of \$5. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good X at the rate of \$4 per unit, but the tax is levied only on the first twenty units purchased. Draw and appropriately label the new budget constraint.

$$\frac{400 - 20\cdot(2+4)}{5} = 56$$

$$\frac{400 - 20(2+4)}{2} = 140$$

$$\begin{array}{r} 140 \\ +\ 20 \\ \hline 160 \end{array}$$





3. [5marks] Wei has $1,600 in income and is considering spending it on one of two lotteries. Lottery A gives Wei a 40% probability of $2,500 in final income and a 60% probability of $900 in final income. Lottery B gives Wei a 30% probability of $625 in final income and a 70% probability of $2,500 in final income. Wei is risk averse with a utility of income function given by $U(I) = I^{1/2}$, where I is her income. Will Wei prefer to keep her $1,600 or will she choose one of the lotteries? Explain.

$$U(L_A) = 0{,}4\sqrt{2500} + 0{,}6\sqrt{900} = 38$$

$$U(L_B) = 0{,}3\sqrt{625} + 0{,}7\sqrt{2500} = 42{,}5$$

$$U(1600) = \sqrt{1600} = 40$$

Wei would prefer L_B : $42.5 > 40 > 38$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Inge consumes two goods, X and Y, according to the utility function $U(X,Y)=X\,Y^{1/2}$. Inge has an income, I, and faces prices for the two goods given by P_X and P_Y.

   a) [5 marks] Derive Inge's demand functions for the goods X and Y.

$$\max\ X Y^{1/2} \quad s.t\ P_x X + P_y Y = I$$

$$MRS = P_x / P_y$$

$$\frac{Y^{1/2}}{\frac{1}{2}\frac{X}{Y^{1/2}}} = \frac{P_x}{P_y} \iff \frac{2Y}{X} = \frac{P_x}{P_y}$$

$$\iff X = \frac{2Y P_y}{P_x}$$

$$P_x \frac{2Y P_y}{P_x} + P_y Y = I$$

$$Y(3P_y) = I$$

$$Y^* = \frac{I}{3P_y} \qquad X^* = \frac{2}{3}\frac{I}{P_x}$$

   b) [5 marks] Assume that Inge's income is $1200, the price of X is $6 and the price of Y is $3. Calculate her demand for each good. What is the elasticity of demand for X at this bundle?

$$Y^* = \frac{I}{3P_y} = \frac{1200}{3 \cdot 3} = 133,\overline{3}$$

$$X^* = \frac{2I}{3P_x} = \frac{2 \cdot 1200}{3 \cdot 6} = \frac{2400}{18} = 133,\overline{3}$$

$$\varepsilon = \frac{\partial X}{\partial P} \cdot \frac{P}{X} = \frac{-2}{3}\frac{I}{P_x^2} \cdot \frac{P_x}{X} = \frac{-2}{3}\frac{I}{P_x \cdot X} = \frac{-2}{3} \cdot \frac{1200}{6 \cdot 133,3}$$

$$= -1$$

c) [5 marks]Suppose the price of X increases to \$8. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

$$X^* = \frac{2 \cdot 1200}{3 \cdot 8} = 100 \qquad y^* = 133,\overline{3} \Rightarrow \text{final bundle}$$

Looking for decomposition bundle:

$U = 1540$ from $X^*_{init} = 133,\overline{3}$ $\quad y^*_{init} = 133,\overline{3}$

We know that $Y = \frac{x P_x}{2P_y}$, so $x\left(\frac{x P_x}{2P_y}\right)^{1/2} = 1540$

$$x^{3/2}\left(\frac{8}{6}\right)^{1/2} = 1540$$

$$x^{3/2}\left(\frac{4}{3}\right)^{1/2} = 1540$$

$$x \approx 121,$$

Substitution effect

$133,\overline{3} - 121 = 12,\overline{3}$

Income effect

$121 - 100 = 21$

2. Jesse has 126 hours per week to divide between leisure, R, and work. When she works, Jesse earns \$24 per hour. She values both leisure and consumption, C, according to the utility function U(R,C)=Min{12R ; C}. The price of the consumption good is unity.

a) [5marks] Derive Jesse's optimal bundle. How much does she work?

Optimality : $12R = C$

B.C : $(126 - R) \cdot 24 = C$

$(126 - R)24 = 12R$

$(126 - R)2 = R$

$252 = 3R$

$R^* = 84$

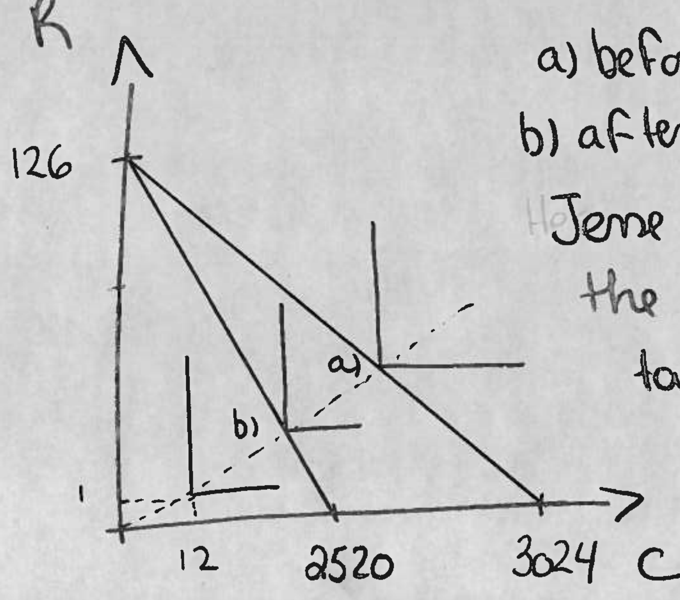
$C^* = 1008$

She will work

$126 - R^* = \text{work}^*$

$42 = \text{work}^*$

b) [5 marks] Explain using a diagram how Jesse's allocation of time between work and leisure changes if she is taxed on her earnings at the rate of $4 per hour. (Assume that her wage decreases by the full $4)



a) before tax

b) after tax

Jesse works more after the introduction of the tax

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c) Starting from the solution to part (a), assume Jesse's boss tells her that new rules mean she must work 48 hours per week. Calculate Jesse's utility level under the new rules and show that she is worse off because of the new rules.

Jene would like to work less than 48 hrs, so she will work exactly 48 hrs if she is forced to

$C = 48 \cdot 24 = 1152$

$U = \min\{12(126-48), 1152\}$

$= \min\{936, 1152\} = 936$

Optimal $U^* = \min\{12 \cdot 84, 1008\}$

$= \min\{1008, 1008\}$

$= 1008$

$936 < 1008$

3. [5 marks] Emily works in the present period and earns an income of $8,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)= C_P^{1/2}C_F^{1/2}$. Emily's savings earn an interest rate of 80%.

a) Derive Emily's optimal consumption bundle and her level of savings.

BC : $C_p + \frac{C_F}{1,8} = 8\ 000\ 000$

MRS : $\dfrac{\frac{1}{2}\frac{C_F^{1/2}}{C_p^{1/2}}}{\frac{1}{2}\cdot\frac{C_p^{1/2}}{C_F^{1/2}}} \Rightarrow \frac{C_F}{C_p} = 1,8$

$C_F = 1,8C_p$

$2C_p = 8\ 000\ 000$

$C_p^* = 4\ 000\ 000$

$C_F^* = 7\ 200\ 000$

b) Suppose that Emily had treated present and future consumption as perfect substitutes rather than as dictated by her Cobb-Douglas tastes. Write an equation for Emily's preferences (perfect substitutes) that would lead Emily to consume all of her present income in the future period (no, she will not starve or die if she has zero present consumption).

$U(C_p, C_F) = C_p + C_F$

MRS = 1

Corner solution

Consume everything in the present

$U_p = 8\,000\,000$

Consume everything in the future

$U_f = 14\,400\,000$

$$U_f > U_p$$

c) Explain and illustrate how Emily's original budget line would change if the government taxed both her present earnings at the rate of 20% and the interest earned on her savings at the rate of 10%.